Thermopower of an infinite Luttinger liquid

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The thermopower of a Luttinger liquid (LL), originating from the energy dispersion of electrons at the Fermi level and/or from the backscattering of electrons by impurities, is analytically evaluated. It is shown that in both cases the thermopower is described by a corresponding Fermi-liquid formula renormalized by an interaction-dependent factor. For an infinite LL the renormalization coefficients decrease with an increase of the electron-electron interaction. In a realistic situation, when a LL wire is connected to leads of noninteracting electrons, the dispersion-induced thermopower in the limit of strong repulsive interaction is strongly suppressed, $S_W^{(d)} \sim g^2 S_0^{(d)} \ll S_0^{(d)}$ (here $S_0^{(d)}$ is the corresponding Fermi-liquid value for the thermopower and $g \ll 1$ is the LL correlation parameter), while the impurity-induced thermopower $S_W^{(i)} \sim S_0^{(i)}/g$ is enhanced by the interelectron interaction.

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I. INTRODUCTION

The transport properties of a Luttinger liquid (LL) differ strongly from those known for the Fermi liquid,[1,2] with one of the most interesting examples pertaining to charge transport through an impurity. In a Fermi liquid (FL) the transmission amplitude of an electron through a local defect (impurity) is fully determined by the shape of the impurity potential and it is a common assumption for metals that this amplitude is a smooth function of the electron energy near the Fermi energy (E_F), i.e., $\varepsilon \sim E_F$. In contrast, the tunneling of electrons in a LL depends primarily on the properties of the LL wire and it is strongly suppressed by the repulsive electron-electron interaction.[3] A simple physical explanation of this effect can be obtained, for instance, in the limit of weak interaction. It was shown[4] that for a weakly interacting one-dimensional (1D) electron system the bare electron transmission amplitude is renormalized by the interelectron interaction and that it vanishes in the vicinity of the Fermi energy, i.e., $t_R(\varepsilon \to E_F) \to 0$. Consequently, the temperature strongly affects the tunneling of a charge in a LL. As a result, the conductance of a LL wire scales with the temperature as a power-law function [for instance, for spinless electrons[3] $G(T) \propto T^{2(g^{-1}-1)}$, where g is the correlation parameter of the LL], whereas for a wire of noninteracting electrons the conductance does not depend on temperature at low temperatures.

If the temperatures of the leads attached to a wire are different, an electric current is induced by the voltage drop and by the temperature gradient across the system. In a LL wire both contributions to the current are power-law functions of the temperature with interaction-dependent exponents. However, the ratio of the corresponding kinetic coefficients, i.e., the thermopower, is affected less by the interaction; that is, it remains a linear function of the temperature as in the case of noninteracting electrons[5,6] (see also Ref. 7 for a calculation of the thermopower using the Hubbard model).

For noninteracting particles the thermopower coefficient $S_0(T)$ is conventionally described, in the linear regime, by Mott's formula as a logarithmic derivative of the conductance G_0 evaluated at the Fermi energy (see, e.g., Ref. 8):

$$S_0(T) \simeq -\frac{\pi^2}{3}\frac{k_B^2 T}{e}\left(\frac{\partial \ln G_0(\varepsilon)}{\partial \varepsilon}\right)_{\varepsilon=\varepsilon_F}. \tag{1}$$

In a previous paper[6] we have shown via a phenomenological model that for a LL connected to leads of noninteracting electrons the thermopower can be represented by a Mott-like formula with an additional interaction-dependent renormalization factor. Although the assumptions made in Ref. 6 are reasonable and the model gives a simple and qualitatively correct description of charge transport in LL wires, there is still no consistent quantitative theory of thermoelectric effects in LL's. In this paper we consider an *infinite* LL and evaluate analytically the thermopower induced (i) by the nonlinearities in the electron spectrum and (ii) by backscattering of the electrons from a single impurity.

It is physically evident that for an ideal (impurity-free) LL the thermopower coefficient is zero.[5] This is a direct consequence of the *linear* spectrum of electrons in the Tomonaga-Luttinger model. Nonlinear corrections to the electron spectrum in the energy region $\varepsilon \sim E_F$ will induce a finite thermopower coefficient.[5] The corrections are small, and they can be treated by perturbation theory. Thus one can readily estimate in this case the dependence of the thermopower on the interaction strength by using simple dimensional analysis, as shown below.

Let us consider first the limit of strong repulsive interaction, $g = v_F/s \ll 1$, where s is the velocity of the plasmons in a LL. In the conventional definition of the thermopower coefficient it is expressed as the ratio of two kinetic coefficients, i.e., $S=-G_{\Delta T}/G_V$, that express the electric conductance due to the voltage drop (V) and due to the temperature difference (ΔT) across the system; that is, the electric current through the system is given by $J=G_V V+G_{\Delta T}\Delta T$. For an ideal infinite LL the conductance is $G_V=(e^2/h)g$ (see Refs. 9 and 3). The cross coefficient (thermoelectric conductance) $G_{\Delta T}$ is fully determined by the nonlinearities in the electron spectrum. The first (quadratic) correction to the linear electron spectrum at $\varepsilon\sim E_F$ is proportional to $\partial v_F^2/\partial E_F$. Therefore, to the lowest order in perturbation theory $G_{\Delta T}\propto\partial v_F^2/\partial E_F$, and in order to restore the correct dimension of $G_{\Delta T}$ (it is dimensionless in the units $e=\hbar=k_B=1$) we have to compensate the dimension of the coefficient of the spectral nonlinearity by the factor T/s^2 (T is the average temperature). This yields the following estimate for the dispersion-induced thermopower of a LL in the limit of strong interaction, $S_L^{(d)}(T,g)\simeq(k_B/e)(gk_BT/v_F)(\partial v_F/\partial E_F)$. In Sec. II we prove the validity of these simple considerations (i) by making use of scaling arguments and (ii) by explicit calculations of $S_L^{(d)}(T,g)$ through perturbation theory.

The thermopower of a LL can be induced also by the backscattering of the electrons by impurities in the wire. For repulsive interaction the potential that causes the backscattering of charged excitations is a relevant perturbation[3] in a LL. Under such circumstances and at low temperatures, one can replace the impurity potential by a weak link (junction) between two semi-infinite segments of a LL wire, and the charge transport through the junction can be evaluated perturbatively by making use of the tunneling Hamiltonian method. To calculate the thermopower of a LL with an impurity, we begin with the general formula for the tunnel current in an interacting electron system (see, e.g., Ref. 10). First, we derive analytical expressions for the finite-temperature spectral densities in a LL with free boundaries; in the literature, the *spectral density function* of a LL is known only in the zero-temperature limit,[11–13] although finite-temperature expressions for the density of states have been known and were used in a number of papers for the evaluation of the I-V characteristics of a LL (see, e.g., Ref. 14). With the help of these formulas we evaluate the kinetic coefficients G_V and $G_{\Delta T}$ and show that electron-electron interactions renormalize the thermopower coefficient multiplicatively. The renormalization factor for an *infinite* LL decreases with an increase of the interaction strength. At first glance this statement seems to contradict our previous result (see Ref. 6), where an enhancement of the thermopower was predicted for a LL wire connected to leads of noninteracting electrons. However, the two problems under study are not identical. In the case of a strong impurity one has to take into account the "Coulomb blockade" effect in the wire to relate the thermopower coefficient $S_L^{(i)}$, calculated for an infinite LL, to the one, $S_W^{(i)}$, for a wire connected to leads of noninteracting electrons. We show that for the last case the thermopower coefficient is enhanced by the interaction.

The paper is organized as follows. In Sec. II the dispersion-induced thermopower is evaluated analytically by (i) simple scaling arguments and (ii) by perturbation theory. In Sec. III the tunnel Hamiltonian method is applied to the calculation of the thermopower coefficient in a LL. In Sec. IV we summarize the main results of the paper and reformulate them for the case of a finite LL wire connected to leads of noninteracting electrons. Technical details of the calculations are given in the appendixes.

II. DISPERSION-INDUCED THERMOPOWER

Let us consider an infinite 1D system of interacting electrons. We will take into account the nonlinear (quadratic) correction to the linearized electron spectrum at $\varepsilon\sim E_F$ (in the following we put $\hbar=k_B=1$):

$$E_r(k)=v_F r(k-k_F)+A(k-k_F)^2, \tag{2}$$

where the index $r=\pm1$ denotes the right (R: $r=+1$) and the left (L: $r=-1$) branches of the 1D electron spectrum, and $A=(v_F/2)(\partial v_F/\partial E_F)$ is the nonlinearity parameter which is assumed to be small, $A\ll v_F/p_F$.

The Hamiltonian of the system takes the form

$$H=H_{0f}+H_{\text{int}}=\sum_r\int dx\,\Psi_r^\dagger(x)\epsilon_r(\nabla_x)\Psi_r(x)$$
$$+\frac{1}{2}\sum_{r,r'}\int dx\int dx'\,U_{r,r'}(x-x')\rho_r(x)\rho_{r'}(x'). \tag{3}$$

Here $\epsilon_r(\nabla_x)=-iv_F r\nabla_x-A(\nabla_x)^2$ is the electron energy operator, $\rho_r(x)=\Psi_r^\dagger(x)\Psi_r(x)$ is the electron density operator, and $U_{r,r'}(x-x')$ is the interaction potential, which we assume in the following to be short ranged, i.e., $U(x)=u_0\delta(x)$.

The Hamiltonian given in Eq. (3) with $A=0$ in the bosonic representation is the standard Hamiltonian of a spinless LL (see, e.g., Ref. 2):

$$H_0=\frac{\pi s}{2}\int_{-\infty}^{\infty}dx\left[g\rho_J(x)\rho_J(x)+\frac{1}{g}\rho_N(x)\rho_N(x)\right]. \tag{4}$$

Here $s=v_F/g$ is the velocity of plasmons and $g^{-1}=\sqrt{1+u_0/\pi v_F}$ is the correlation parameter for spinless electrons. The density operators $\rho_{N(J)}(x)=\rho_R(x)\pm\rho_L(x)$ obey the anomalous commutation relation $[\rho_N(x),\rho_J(x')]=-(i/2\pi)\partial_x\delta(x-x')$.

The dispersion of the electron spectrum results in unharmonic bosonic terms[15] that describe the interaction of plasmons in the LL:

$$H_A=A\frac{\pi^2}{12}\int_{-\infty}^{\infty}dx[\rho_N^3(x)+3\rho_N(x)\rho_J^2(x)]+\text{H.c.} \tag{5}$$

It is useful to reexpress the Hamiltonians in Eqs. (4) and (5) in terms of bosonic fields $\Phi(x)$ and $\Pi(x)$, obeying the canonical commutation relations

$$\rho_N=\frac{1}{2\pi}\partial_x\Phi,\quad \rho_J=-2\Pi=-\frac{1}{2\pi sg}\partial_t\Phi,$$
$$[\Pi(x),\Phi(x')]=-i\delta(x-x'). \tag{6}$$

Then the Hamiltonian of our model takes the form (see also Ref. 15)

$$H=\frac{1}{8\pi sg}\int_{-\infty}^{+\infty}dx[(\partial_t\Phi)^2+s^2(\partial_x\Phi)^2]$$
$$+\frac{A}{96\pi}\int_{-\infty}^{+\infty}dx\left[(\partial_x\Phi)^3+\frac{3}{(sg)^2}(\partial_x\Phi)(\partial_t\Phi)^2+\text{H.c.}\right]. \tag{7}$$

In the linear response approximation the average dc current

$$J(x)=\int dx'\sigma^{(1)}(x,x')E(x')+\frac{1}{T}\int dx'\sigma^{(2)}(x,x')\nabla T(x') \tag{8}$$

is determined by two kinetic coefficients $\sigma^{(1),(2)}$, which, according to the Kubo formalism[16] (see also Ref. 17), can be expressed through the current-current and energy-current correlation functions

$$\sigma^{(1)}(x,x')=\int_0^\infty dt\int_0^\beta d\lambda\langle j(-i\lambda,x)j(t,x')\rangle,$$

$$\sigma^{(2)}(x,x')=\int_0^\infty dt\int_0^\beta d\lambda\langle q(-i\lambda,x)j(t,x')\rangle, \tag{9}$$

where $\beta\equiv 1/T$ is the inverse temperature, $j=-(e/2\pi)\partial_t\Phi$ is the charge current operator, and $q(x)$ is the energy current operator defined by the continuity equation $\partial_t h(x)+\partial_x q(x)=0$ with the Hamiltonian density $h(x)$ given as $H_0=\int dx\, h(x)$. It is easy to find that in terms of the density operators the energy current takes the form

$$q(x)=\frac{\pi s^2}{2}\rho_J(x)\rho_N(x)+\text{H.c.}=-\frac{s}{8\pi g}\partial_t\Phi\,\partial_x\Phi+\text{H.c.} \tag{10}$$

[Actually, the regularized density operators always commute at equal points $x=x'$ (see Ref. 15) and we do not have to care about the order of operators in Eqs. (5), (7), and (10).]

In a homogeneous system the kinetic coefficients do not depend on the coordinate, i.e., $\sigma^{(1)}(L\rightarrow\infty)\equiv G_V$ and $T^{-1}\sigma^{(2)}(L\rightarrow\infty)\equiv G_{\Delta T}$, and the current depends only on the integral quantities of the system, that is, the bias voltage (V) and the temperature difference (ΔT). The thermopower coefficient S is given simply by the ratio of the two transport coefficients:

$$S|_{J=0}=\frac{V}{\Delta T}=-\frac{G_{\Delta T}}{G_V}. \tag{11}$$

Since the electron dispersion at the Fermi energies is weak $[p_F(\partial v_F/\partial E_F)\ll 1]$, we can evaluate the thermopower coefficient perturbatively. To lowest order in perturbation theory with respect to the nonlinearity coefficient (A), one gets

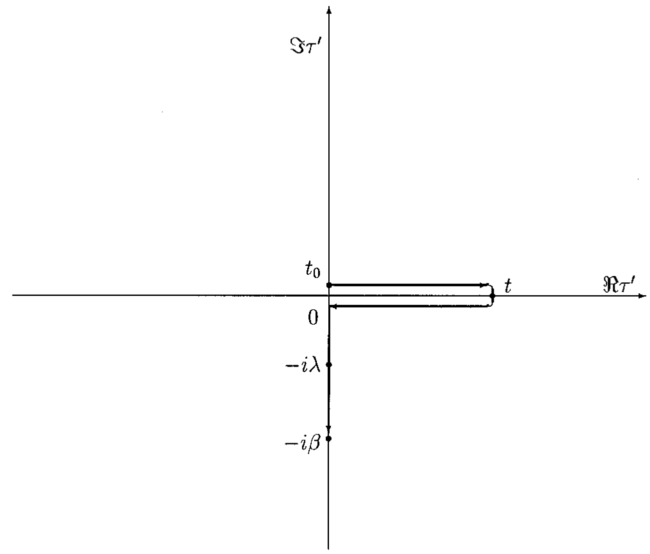

FIG. 1. The integration contour C in the complex time plane.

$$\sigma^{(1)}(x,x')\simeq\int_0^\infty dt\int_0^\beta d\lambda\langle\hat{T}_c j(-i\lambda,x)j(t,x')\rangle_0, \tag{12}$$

$$\sigma^{(2)}(x,x')\simeq\pi e s^3 g\int_0^\infty dt\int_0^\beta d\lambda\langle\hat{T}_c q(-i\lambda,x)j(t,x')$$
$$\times S_1(-i\beta,0)\rangle_0, \tag{13}$$

where $S_1(-i\beta,0)=-\int_C\hat{H}_A(\tau')d\tau'$ and $\hat{H}_A(\tau')$ is the nonharmonic part of the Hamiltonian, Eq. (7), in the interaction representation. The thermal average $\langle\cdots\rangle_0$ is taken with respect to the unperturbed Hamiltonian, Eq. (4), and the symbol $\hat{T}_c$ denotes the ordering of operators along the contour C in the complex time plane (see Fig. 1). The correlators in Eqs. (12) and (13) can be rewritten in terms of the density operators $\rho_{N,J}$, which have known correlation functions (see Appendix A).

It is evident that to lowest order in perturbation theory the conductance does not depend on the nonlinear contributions $G_V=ge^2/2\pi$ (see Refs. 9 and 3). The calculation of $G_{\Delta T}$ is straightforward, although quite lengthy, and it is outlined in Appendix A. Here we show how to determine in perturbation theory the dependence of the thermopower on the correlation parameter g without explicit calculations.

Let us consider the canonical transformation

$$\Phi(x,t)=\sqrt{g}\chi,\quad \Pi_\Phi=\frac{1}{\sqrt{g}}\Pi_\chi, \tag{14}$$

which allows us to eliminate the factor g in the harmonic part of the LL Hamiltonian, Eq. (7). The current operators j and q in the correlation functions can be reexpressed in terms of the current operators $j^{(0)}=-(e/2\pi)\partial_t\chi$ and $q^{(0)}=-(s/4\pi)\partial_x\chi\partial_t\chi$ for "noninteracting particles," as follows:

$$j=-\frac{e}{2\pi}\partial_t\Phi=\sqrt{g}j^{(0)},\quad q=-\frac{s}{4\pi}\partial_x\Phi\partial_t\Phi=q^{(0)}. \tag{15}$$

Now the structure of the correlation functions is obvious:

$$G_V\simeq g\int_0^\infty dt\int_0^\beta d\lambda\langle\hat{T}_c j^{(0)}(-i\lambda)j^{(0)}(t)\rangle_0=\frac{ge^2}{2\pi}, \tag{16}$$

$$\begin{aligned}G_{\Delta T}&\simeq\frac{\sqrt{g}}{T}\int_0^\infty dt\int_0^\beta d\lambda\langle\hat{T}_c q^{(0)}(-i\lambda)j^{(0)}(t)S_1(-i\beta)\rangle_0\\&=A(g^2+C_1)W(s,T),\end{aligned} \tag{17}$$

where C_1 is a numerical constant (its value can be determined by a perturbation analysis: see Appendix A). Notice that the function $W(s,T)$ does not depend explicitly either on the Fermi velocity or on the correlation parameter g. The dependence of W on the plasmon velocity and temperature can be found by considering the limit $g=1$, where the analytic expression for the dispersion-induced thermopower is known (see, e.g., Ref. 18):

$$S_F(T)=-\frac{\pi^2}{3}\frac{T}{ev_F}\frac{\partial v_F}{\partial E_F}. \tag{18}$$

From Eqs. (11), (16), (17), and (18) one readily gets

$$W=\frac{\pi}{3}\frac{eT}{s^2}\frac{1}{(1+C_1)}. \tag{19}$$

Thus, by making use of dimensional analysis supplemented with scaling arguments, one can get (up to a numerical constant) the expression for the dispersion-induced thermopower:

$$S_L^{(d)}(T,g)=-\frac{\pi^2}{3}\frac{g(g^2+C_1)}{1+C_1}\frac{T}{ev_F}\frac{\partial v_F}{\partial E_F}. \tag{20}$$

In Appendix A it is shown, by a direct calculation of the correlation functions in perturbation theory, that $C_1=1$, and our final result for the dispersion-induced thermopower is

$$S_L^{(d)}(T,g)=\frac{g(g^2+1)}{2}S_F(T). \tag{21}$$

It follows from Eq. (21) that the repulsive electron-electron interaction ($g<1$) suppresses the thermopower of a homogeneous LL. For strongly interacting electrons (i.e., for $g\ll 1$) the above result [Eq. (21)] coincides with the simple estimate $S_L^{(d)}\sim gS_F$ presented in the introductory section of the paper. In this limit our result is consistent with the expression given in Eq. (8) of Ref. 5 [we believe that the appearance of the factor g in the denominator of Eq. (8) in Ref. 5 is a misprint]. Note that in Ref. 5 a somewhat different perturbation Hamiltonian was used: namely, the first term in our Hamiltonian [see Eq. (5)]. Actually, the form of the dispersion-induced terms in a LL theory is not universal [in a renormalization group (RG) sense]. We started with Eq. (5) [see also Eq. (5.3) in Ref. 15], where the bare couplings are determined by the chosen form of the electron energy dispersion (i.e., the energy-momentum relation). The operators in the perturbation Hamiltonian, Eq. (5), are (formally) ‘‘irrelevant’’ and thus the perturbation calculation is justified. However, in a RG analysis other terms with the same scale dimension ($d=3$) could be generated by the loop corrections. Consequently, in Eq. (21) only terms of leading order in $g\ll 1$ can be trusted from scaling arguments. We left in this formula the full dependence on g because it reproduces the correct result for noninteracting electrons ($g=1$) and represents a possible scenario of thermopower crossover from repulsively interacting 1D electrons ($g<1$) to a LL with a bulk attraction ($g>1$).

III. THERMOPOWER OF AN INFINITE LUTTINGER LIQUID WITH AN IMPURITY

In a gapless 1D electron system the effects induced by the nonlinearities of the electron spectrum around the Fermi energy are weak, and as we showed in the previous section they are suppressed further by the repulsive interaction [roughly by a factor $(v_F/s)^2$ in the transport coefficients]. Drastic changes in the properties of a LL are caused by local impurity potentials.

The conductance G_V of an infinite LL with an impurity was calculated for the first time in Ref. 3, where it has been shown that G_V scales with the temperature as a power-law function with an exponent that depends strongly on the coupling constant. It is natural to assume an analogous behavior for the thermal-electric coefficient $G_{\Delta T}$ (we will prove this assumption later). Thus, from purely dimensional considerations, one could expect[5] a linear-T behavior of the ‘‘impurity-induced’’ thermopower $S_L^{(i)}(T,g)$ even for strongly interacting systems. However, unlike the case of dispersion-induced thermopower, the dependence of $S_L^{(i)}$ on the dimensionless parameter g cannot be obtained from such dimensional analysis.

In this section we evaluate analytically the current induced by the temperature difference, $J_{\Delta T}$, and determine the dependence of the thermopower of a LL on the interaction strength. Since it is well known that even a weak bare potential strongly suppresses at low temperatures the transport of a charge in a LL, it is reasonable to consider, from the very beginning, the case of weak electron tunneling, for which the problem can be solved by a tunneling Hamiltonian method.

We start with a general expression for the tunnel current in a system of interacting electrons (see, e.g., Ref. 10):

$$\begin{aligned}J=2\pi e\int_{-\infty}^{+\infty}dp_1\int_{-\infty}^{+\infty}dp_2|\hat{T}_{p_1,p_2}|^2\int_{-\infty}^{\infty}d\varepsilon\, A_{T_1}(p_1,\varepsilon)\\\times A_{T_2}(p_2,\varepsilon+eV)[f_{T_2}(\varepsilon+eV)-f_{T_1}(\varepsilon)],\end{aligned} \tag{22}$$

where $f_T(\varepsilon)=[\exp\{\varepsilon/T\}+1]^{-1}$ is the Fermi-Dirac distribution function, $A_T(p,\varepsilon)$ is the electron spectral density, and

$\hat{T}_{p_1,p_2}$ is the bare tunneling amplitude. We assume (see, e.g., Ref. 19) that the tunnel probability $|\hat{T}|^2$ depends on the momentum of only one of the two segments of the 1D wire (say, for definiteness "1") and that it is a smooth function of the momentum for $p \sim p_F$. Under this assumption we may write

$$|\hat{T}_{p_1,p_2}|^2 \Rightarrow |\hat{T}_{r_1,r_2}(q_1)|^2 \simeq \delta_{r_1,r_2}\left(t_S^2 + \frac{\partial t_S^2}{\partial E_F} v_F r_1 q_1\right) + \delta_{r_1,-r_2}\left(t_R^2 + \frac{\partial t_R^2}{\partial E_F} v_F r_1 q_1\right). \tag{23}$$

Here $p_m = r_m p_F + q_m$, $m=1,2$, $r_m = \pm 1$, and the parameters $t_{S,R}^2 \equiv t_{S,R}^2(E_F)$ will be specified later.

In the linear response approximation the tunnel current is a sum of two currents, $J = J_V(T) + J_{\Delta T}(T)$: one induced by the voltage drop V across the junction and the other induced by temperature difference ΔT between two segments of the wire, i.e.,

$$J_V = 2\pi e^2 V \sum_{r_1,r_2} \int_{-\infty}^{\infty} dq_1 \int_{-\infty}^{\infty} dq_2 \int_{-\infty}^{\infty} d\varepsilon |\hat{T}_{r_1,r_2}(q_1)|^2 \times A_T(r_1,q_1,\varepsilon) A_T(r_2,q_2,\varepsilon) \frac{\partial f(\varepsilon)}{\partial \varepsilon}, \tag{24}$$

$$J_{\Delta T} = 2\pi e \Delta T \sum_{r_1,r_2} \int_{-\infty}^{\infty} dq_1 \int_{-\infty}^{\infty} dq_2 \int_{-\infty}^{\infty} d\varepsilon |\hat{T}_{r_1,r_2}(q_1)|^2 \times A_T(r_1,q_1,\varepsilon) A_T(r_2,q_2,\varepsilon) \frac{\partial f(\varepsilon)}{\partial T}, \tag{25}$$

where now the special functions are taken at a mean temperature T and the derivatives of the distribution function are given as

$$\frac{\partial f(\varepsilon)}{\partial \varepsilon} = -\frac{1}{4T} \frac{1}{\cosh^2\left(\frac{\varepsilon}{2T}\right)}, \quad \frac{\partial f(\varepsilon)}{\partial T} = \frac{\varepsilon}{4T^2} \frac{1}{\cosh^2\left(\frac{\varepsilon}{2T}\right)}. \tag{26}$$

To evaluate the kinetic coefficients G_V and $G_{\Delta T}$, one needs to know the exact analytic expression for the spectral function $A_T(q,\varepsilon)$ at a finite temperature. By definition,

$$A_{T_m}(r_m, q_m, \omega) = -\frac{1}{\pi} \mathrm{Im}[G_{r_m}^R(q,\omega)], \tag{27}$$

where $G_{r_m}^R(q,\omega)$ is the Fourier transform of the retarded Green's function:

$$G_{r_m}^R(x,t) = -i\Theta_H(t)\langle\{\Psi_{r_m}(x,t), \Psi_{r_m}^\dagger\}_+\rangle = \Theta_H(t)[G_{r_m}^>(x,t) - G_{r_m}^<(x,t)]. \tag{28}$$

Here $\Theta_H(t)$ is the Heaviside step function and the standard definitions for $G^>$ and $G^<$ (see, e.g., Ref. 10, Sec. 3.7) are adopted. To calculate these Green's functions we will use a bosonization technique. The conventional procedure (see, e.g., Ref. 2) is to represent the fermion field operator $\Psi(x,t)$ as an exponential of the boson fields $\Phi(x,t)$ and $\Theta(x,t)$ with the commutation relations $[\Theta(x), \Phi(x')] = 2\pi i\, \mathrm{sgn}(x-x')$, yielding

$$\Psi_{m,r_m}(x,t) = \frac{1}{\sqrt{2\pi a}} U_{m,r_m}^\dagger \exp\left\{-\frac{i}{2}[r_m \Phi_m(x,t) + \Theta_m(x,t)]\right\}. \tag{29}$$

Here a is a cutoff parameter ($a \sim v_F/E_F$) and $U_{m,r_m}^\dagger$ is a unitary raising operator which increases the number of electrons on the branch r_m by one particle, but does not affect the bosonic excitations. For our purpose its specific form is irrelevant.

If we neglect tunneling we are dealing with two semi-infinite LL's with an open boundary which reflects the electrons perfectly. It is helpful to formulate the corresponding boundary condition in terms of mirror images; then,[20] $\Psi_{L,m}(x) = -\Psi_{R,m}(-x)$. The boson fields yielding this boundary condition in the momentum representation take the form (see Appendix B)

$$\Theta_m(x) = i\int_{-\infty}^{\infty} dp \sqrt{\frac{2s}{g\epsilon_p}} (b_p - b_p^\dagger) \cos\left(\frac{\epsilon_p}{s} x\right),$$

$$\Phi_m(x) = \int_{-\infty}^{+\infty} dp \sqrt{\frac{2sg}{\epsilon_p}} (b_p + b_p^\dagger) \sin\left(\frac{\epsilon_p}{s} x\right), \tag{30}$$

where b_p and $b_p^\dagger$ are the standard bosonic annihilation and creation operators ($[b_p, b_{p'}^\dagger] = \delta_{p,p'}$) and $\epsilon_p = s|p|$ is the energy of the bosonic excitation with momentum p.

With the help of Eqs. (29) and (30) it is straightforward to evaluate the fermion Green's functions. In particular, for $iG^>$ one gets in the vicinity of the contact ($x \sim 0$) the following expression:

$$\langle \Psi_{m,r_m}(x,t) \Psi_{m,r_m}^\dagger \rangle \simeq \frac{1}{2\pi a} \left[\frac{1}{\left(1 + i\frac{v_F \chi}{a}\right)} \frac{\pi T_m \chi}{\sinh(\pi T_m \chi)}\right]^{(1/g + r_m)/2} \times \left[\frac{1}{\left(1 + i\frac{v_F \eta}{a}\right)} \frac{\pi T_m \eta}{\sinh(\pi T_m \eta)}\right]^{(1/g - r_m)/2}, \tag{31}$$

where $\chi = t - x/s$ and $\eta = t + x/s$.

The next step is to calculate the Fourier transform of the Green's functions. It is helpful now to introduce new variables $X_\pm = \pi T(t \pm x/s)$ and $\Omega_\pm = (\omega \pm ks)/2$ and the dimensionless temperature $\overline{T} = \pi T a/v_F$. In terms of these variables the Fourier transform of $iG^>$ has the form

$$iG^{>}_{r_m}(\Omega_+,\Omega_-)=\frac{\overline{T}^{1/g-1}}{8\pi^2 gT}\exp\left(-i\frac{\pi}{2}\frac{1}{g}\right)\int_{-\infty}^{\infty}dX_+\int_{-\infty}^{\infty}dX_-$$
$$\times\exp\left[\frac{i}{\pi T}(\Omega_+X_-+\Omega_-X_+)\right]\left[\frac{1}{X_--i\overline{T}}\frac{X_-}{\sinh X_-}\right]^{(1/g+r_m)/2}\left[\frac{1}{X_+-i\overline{T}}\frac{X_+}{\sinh X_+}\right]^{(1/g-r_m)/2}. \tag{32}$$

The spectral density $A(\omega,q)$ is expressed through $G^{>}(\omega,q)$ by the standard relation

$$A_{r_m}(\Omega_+,\Omega_-)=\frac{1}{2\pi}\mathrm{Im}[iG^{>}_{r_m}(\Omega_+,\Omega_-)+iG^{>}_{r_m}(-\Omega_+,-\Omega_-)]. \tag{33}$$

Since we are interested in the limit $\overline{T}\ll 1$, the integrals in Eq. (32) can be taken analytically. After some algebra we get the analytic expression for the spectral density function of a spinless LL with an open boundary, at finite temperatures $T\ll E_F$:

$$A_{r_m}(\Omega_+,\Omega_-)=\frac{1}{(2\pi)^3 gT}\overline{T}^{1/g-1}\cosh\left(\frac{\Omega_++\Omega_-}{2T}\right)\int_{-\infty}^{\infty}dX_-\cos\left(\frac{\Omega_+X_-}{\pi T}\right)\left(\frac{1}{\cosh X_-}\right)^{(1/g+r_m)/2}$$
$$\times\int_{-\infty}^{\infty}dX_+\cos\left(\frac{\Omega_-X_+}{\pi T}\right)\left(\frac{1}{\cosh X_+}\right)^{(1/g-r_m)/2}$$
$$=\begin{cases}\dfrac{1}{(2\pi)^3 gT}(2\overline{T})^{1/g-1}\cosh\left(\dfrac{\Omega_++\Omega_-}{2T}\right)\dfrac{\left|\Gamma\left(\frac{1}{4}\left[\frac{1}{g}+r_m\right]+\frac{i}{2\pi}\beta\Omega_+\right)\right|^2}{\Gamma\left(\frac{1}{2}\left[\frac{1}{g}+r_m\right]\right)}\dfrac{\left|\Gamma\left(\frac{1}{4}\left[\frac{1}{g}-r_m\right]+\frac{i}{2\pi}\beta\Omega_-\right)\right|^2}{\Gamma\left(\frac{1}{2}\left[\frac{1}{g}-r_m\right]\right)}, & \text{if } g<1,\\ \delta(\omega-r_m v_F k) & \text{if } g=1.\end{cases} \tag{34}$$

Substituting Eq. (34) into Eqs. (24) and (25) and performing the integration over the momenta and energy (see Appendix C), one gets the desired kinetic coefficients (here we restore the normal dimensionality)

$$G_V=\frac{e^2}{2\pi\hbar}t_0^2R_g^{(j)}(T),\quad G_{\Delta T}=\frac{\pi^2}{3}\frac{e}{\hbar}k_B^2T\frac{\partial t_0^2}{\partial E_F}R_g^{(j)}(T), \tag{35}$$

where the renormalization coefficients $R_g^{(j)}(T)$ $(j=1,2)$ are given by

$$R_g^{(j)}(T)=\frac{2j+1}{2}B\left(\frac{2j+1}{2},\frac{1}{g}\right)\left(\pi\frac{k_BTa}{\hbar v_F}\right)^{2(1/g-1)}. \tag{36}$$

Here $B(x,y)=\Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the beta function and the effective transmission probability $t_0^2\ll 1$ at the Fermi energy is defined as

$$t_0^2=\left(\frac{2\pi}{\hbar v_F}\right)^2\sum_{r_1,r_2}(t_S^2\delta_{r_1,r_2}+t_R^2\delta_{r_1,-r_2}). \tag{37}$$

The expression for the conductance given in Eq. (35) coincides with the known result.[3,21] One can see from Eqs. (35) and (36) that the thermoelectric cross coefficient $G_{\Delta T}$ is renormalized by the interaction in analogy with the conductance. Consequently, the influence of the interaction on the thermopower is far less dramatic than that on the transport coefficients. The thermopower of a LL is still a linear function of temperature[5,6] as is the thermopower of a system of noninteracting electrons. The electron-electron interaction in a LL model leads only to a temperature-independent multiplicative renormalization of the thermopower S_0 of the free electrons:

$$S_L^{(i)}(T,g)=\frac{3g}{2+g}S_0(T). \tag{38}$$

For an infinite LL the renormalization factor decreases with increase of the interelectron interaction, and for strongly interacting particles $S_L^{(i)}(g\ll 1)\simeq(3/2)gS_0$.

IV. CONCLUSION

In this paper we have evaluated the thermopower of an infinite spinless LL induced by (i) the dispersion of the electron spectrum near the Fermi energy and by (ii) the backscattering of the electrons by an impurity. We showed that the thermopower treated by perturbation theory (with respect to the nonlinearity of the electronic spectrum and the bare electron tunneling amplitude) is described by the Fermi-liquid formulas renormalized by interaction-dependent factors. For an *infinite* LL the renormalization coefficients, Eqs.

(21) and (38), are decaying functions of the interaction strength $V_0 \sim e^2$, since the correlation parameter is equal to $g^{-1} = \sqrt{1 + V_0/\pi\hbar v_F}$ for spinless electrons.

To explore whether the electron-electron interactions suppresses the thermopower of 1D electron systems, we have solved the problem for an infinite LL. In real experiments the LL wire (e.g., a carbon nanotube[22]) is connected to 3D or 2D metallic leads where the electrons can be regarded as noninteracting particles. It is known that the transport properties of a LL wire connected to (noninteracting) electron reservoir differ from the transport properties calculated for an infinite LL, even for adiabatic contacts. The best known example of such a behavior is the conductance G_L of an impurity-free LL wire. For an infinite LL, formally $G_L = gG_0$ (G_0 is the conductance quantum, $G_0 = e^2/h$ for spinless electrons), while for a LL wire connected to leads, $G_L = G_0$ [the so-called "no renormalization theorem" for the conductance of a LL (Refs. 23–26)]. Note that the heat conductance G_T is also different for the above two situations (see Refs. 27 and 28).

To estimate the thermopower of a finite LL wire adiabatically connected to leads of noninteracting electrons we will follow the approach proposed in Ref. 29. In the case of weak tunneling through the impurity, the voltage drop across the impurity and the one measured between the leads are different quantities. This fact is evident in the limit of strong interaction $g^2 \sim \hbar v_F/e^2 \ll 1$ when the Coulomb blockade is pronounced; the shift of the chemical potentials of the leads $\Delta\mu_L = eU$ cannot change significantly the voltage drop V across the impurity (placed in the middle of a sufficiently long LL wire). In a previous study it has been shown that $V = g^2 U$ for arbitrary interaction strength.[29] Therefore, to relate (at least qualitatively) the thermopower $S_L^{(i)}(T,g)$ evaluated above to the thermopower $S_W^{(i)}(T,g)$ of a LL wire adiabatically connected to leads of noninteracting electrons, we have to replace first the voltage V in our formulas by $g^2 U$. Since this substitution affects only the voltage induced current, it influences the thermopower $S_W^{(i)}(T,g) \simeq S_L^{(i)}(T,g)/g^2$ and now $S_W^{(i)}(T,g) \sim S_0(T)/g \gg S_0(T)$ for strongly interacting particles. We see that in a real situation, when the voltage drop is measured between the leads the electron-electron interaction in the wire enhances the impurity-induced thermopower. It supports our claim[6] based on estimation of the thermopower in a phenomenological model of charge and heat transport in a LL. An explicit calculation of the correlation functions in the presence of the leads remains a subject for future studies.

Finally, we may inquire about the effect of the leads on the dispersion-induced thermopower. In the absence of electron backscattering the leads keep the conductance of a LL wire unrenormalized,[23–25] i.e., $G_L = e^2/h$. Therefore, the dispersion-induced thermopower of a finite LL wire, $S_W^{(d)}(T,g) = gS_L^{(d)}(T,g)$, is suppressed even stronger by the interelectron interaction than the above calculated quantity $S_L^{(d)}$. For strongly interacting ($g \ll 1$) particles $S_W^{(d)}(T,g) \sim g^2 S_F(T) \ll S_F(T)$ [S_F is the corresponding Fermi-liquid thermopower, Eq. (18)]. Thus one could expect that in experiments involving wires of strongly correlated electrons the measured thermopower would be associated mostly with imperfections in the wire (impurities, barriers at the boundaries between the 1D wire and the leads, etc.).

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APPENDIX A

The density operators $\rho_{N,J}(t,x)$ in momentum representation take the form (see, e.g., Ref. 2)

$$\rho_N(x,t) = \frac{N_0}{L} + \frac{1}{2\pi}\int_{-\infty}^{+\infty} dp\, \sqrt{\frac{g\epsilon_p}{s}}[b_p e^{-i(px-\epsilon_p t)} + b_p^\dagger e^{i(px-\epsilon_p t)}], \tag{A1}$$

$$\rho_J(x,t) = \frac{J_0}{L} + \frac{1}{2\pi}\int_{-\infty}^{+\infty} dp\, \sqrt{\frac{\epsilon_p}{sg}}\,\mathrm{sgn}(p)[b_p e^{-i(px-\epsilon_p t)} + b_p^\dagger e^{(px-\epsilon_p t)}], \tag{A2}$$

where L is the size of the system ($L \to \infty$), N_0 is the number of extra (above the Fermi level) electrons, J_0 is the zero-mode current, b_p and $b_p^\dagger$ are the standard bosonic annihilation and creation operators ($[b_p, b_{p'}^\dagger] = \delta_{p,p'}$), and $\epsilon_p = s|p|$ is the energy of bosonic excitation with momentum p.

By making use of Eqs. (A1) and (A2) it is straightforward to calculate the Matsubara Green functions for the density operators ($L \to \infty$):

$$\langle \hat{T}_\tau \rho_N(-i\tau,x)\rho_N(0,y)\rangle \cong -\frac{g}{2\pi s^2\beta}\sum_n e^{i\bar{\omega}_n\tau}\bar{\omega}_n \frac{\cosh\left[\frac{\bar{\omega}_n}{s}\left(x-y-\frac{L}{2}\right)\right]}{\sinh\left[\frac{\tilde{\omega}_n}{s}\frac{L}{2}\right]}, \tag{A3}$$

$$\langle \hat{T}_\tau \rho_J(-i\tau,x)\rho_J(0,y)\rangle \cong -\frac{1}{2\pi s^2 g\beta}\sum_n e^{i\bar{\omega}_n\tau}\bar{\omega}_n \frac{\cosh\left[\frac{\bar{\omega}_n}{s}\left(x-y-\frac{L}{2}\right)\right]}{\sinh\left[\frac{\tilde{\omega}_n}{s}\frac{L}{2}\right]}, \tag{A4}$$

$$\langle \hat{T}_\tau \rho_N(-i\tau,x)\rho_J(0,y)\rangle \cong -\frac{1}{2\pi s^2\beta}\sum_n e^{i\overline{\omega}_n\tau}\overline{\omega}_n \frac{\sinh\left[\frac{\overline{\omega}_n}{s}\left(x-y-\frac{L}{2}\right)\right]}{\sinh\left[\frac{\overline{\omega}_n}{s}\frac{L}{2}\right]}. \tag{A5}$$

Here $\rho_{N,J}\equiv\rho_{N,J}(0,0)$ and $\overline{\omega}_n = i2\pi n/\beta$ is the Matsubara frequency ($\beta = T^{-1}$, $n=0,\pm1,\pm2,\ldots$). One readily gets from Eqs. (A3)–(A5)

$$\langle \rho_N\rho_N\rangle = \frac{g}{\pi s L}\sum_m \frac{\varepsilon_m}{e^{\beta\varepsilon_m}-1},$$

$$\langle \rho_J\rho_J\rangle = \frac{1}{\pi s g L}\sum_m \frac{\varepsilon_m}{e^{\beta\varepsilon_m}-1}, \quad \langle \rho_J\rho_N\rangle = 0. \tag{A6}$$

In perturbation theory the kinetic coefficients can be represented as the time-ordered product of the ρ_N and ρ_J density operators. In particular, for $\sigma^{(2)}$ in the static limit $\omega\to 0$ [see Eq. (13)], one gets

$$\begin{aligned}\sigma^{(2)} = {} & A\frac{e\pi^3 s^3 g}{6}\lim_{\substack{\overline{\omega}\to 0\\ L\to\infty}}\frac{1}{\overline{\omega}}\int_0^\beta d\lambda\int_0^\beta d\tau_1\int_0^L dx_1 \exp(i\lambda\overline{\omega})\\ & \times[\langle \hat{T}_\tau \rho_J(-i\lambda,x)\rho_N(-i\lambda,x)\rho_J(0,x)\\ & \times \rho_N(-i\tau_1,x_1)\rho_N(-i\tau_1,x_1)\rho_N(-i\tau_1,x_1)\rangle\\ & +3\langle \hat{T}_\tau \rho_J(-i\lambda,x)\rho_N(-i\lambda,x)\rho_J(0,x)\\ & \times \rho_J(-i\tau_1,x_1)\rho_J(-i\tau_1,x_1)\rho_N(-i\tau_1,x_1)\rangle].\end{aligned} \tag{A7}$$

Wick's theorem allows us to reduce the time-ordered product of operators to the sum of the product of Green's functions. In our case the thermoelectric coefficient takes the form

$$\begin{aligned}\sigma^{(2)} = {} & A\frac{e\pi^3 s^3 g}{2}\lim_{\substack{\tilde{\omega}\to 0\\ L\to\infty}}\frac{1}{\overline{\omega}}\int_0^\beta d\lambda\int_0^\beta d\tau_1\int_0^L dx_1 \exp(i\lambda\overline{\omega})\\ & \times[\langle \hat{T}\rho_J(-i\lambda,x)\rho_J(0,x)\rangle\langle \hat{T}\rho_N(-i\lambda,x)\\ & \times\rho_J(-i\tau_1,x_1)\rangle](\langle\rho_J\rho_J\rangle + \langle\rho_N\rho_N\rangle),\end{aligned} \tag{A8}$$

where $\overline{\omega} = i\omega$. Substitution of the Green's functions into the last equation yields

$$\sigma^{(2)} = \frac{Ae}{s^2\beta^2}\frac{\pi(g^2+1)}{6}. \tag{A9}$$

APPENDIX B

Here we derive following Ref. 20 the expressions for the momentum representation of the bosonic fields $\Phi(x)$ and $\Theta(x)$ for a LL with an open boundary. The impurity potential at $x=0$ is modeled by the boundary which reflects electrons perfectly. Thus one may regard the LL wire as consisting of two independent (in the absence of tunneling) segments. Let us continue the fermion field $\Psi_{r_m}(x)$ from the segment "1(2)" to the segment "2(1)." The fermion field Ψ must satisfy the condition

$$\Psi_{L,m}(x) = -\Psi_{R,m}(-x) \tag{B1}$$

on each segment $m=1,2$. Hence the densities $\rho_{N,J,L,R}$ and the field operators have to obey the relations $\rho_L(x) = \rho_R(-x)$, $\rho_N(x)=\rho_N(-x)$, $\rho_J(x) = -\rho_J(-x)$, $\Theta(x) = \Theta(-x)$, and $\Phi(x) = -\Phi(-x)$. It is natural to consider that for the case of nonininteracting electrons ($g=1$) the fields Θ and Φ are the stationary waves

$$\Theta_m^{(0)}(x) = i\int_{-\infty}^{+\infty} dp\,\sqrt{\frac{2v_F}{\epsilon_p}}(b_p - b_p^\dagger)\cos\left(\frac{\epsilon_p}{v_F}x\right),$$

$$\Phi_m^{(0)}(x) = \int_{-\infty}^{+\infty} dp\,\sqrt{\frac{2v_F}{\epsilon_p}}(b_p + b_p^\dagger)\sin\left(\frac{\epsilon_p}{v_F}x\right), \tag{B2}$$

where b_p and $b_p^\dagger$ are bosonic annihilation and creation operators ($[b_p, b_{p'}^\dagger] = \delta_{p,p'}$) and $\epsilon_p = v_F|p|$. Substituting Eq. (B2) into the LL Hamiltonian, Eq. (4), we observe that the Hamiltonian is not diagonal in the annihilation and creation operators. It is diagonalized by the Bogoliubov's transformation, and the transformed fields $\Theta_m(x)$ and $\Phi_m(x)$ take the form

$$\begin{aligned}\Theta_m(x) &= \hat{U}\Theta_m^0(x)\hat{U}^{-1}\\ &= i\int_{-\infty}^{+\infty} dp\,\sqrt{\frac{2s}{g\epsilon_p}}(b_p - b_p^\dagger)\cos\left(\frac{\epsilon_p}{s}x\right),\end{aligned} \tag{B3}$$

$$\Phi_m(x) = \hat{U}\Phi_m^0(x)\hat{U}^{-1} = \int_{-\infty}^{+\infty} dp\,\sqrt{\frac{2sg}{\epsilon_p}}(b_p + b_p^\dagger)\sin\left(\frac{\epsilon_p}{s}x\right), \tag{B4}$$

where the unitary operator $\hat{U}$ is

$$\hat{U} = \exp\left(\frac{1}{2}\varphi\int_{-\infty}^{+\infty} dq[b_q^\dagger b_{-q}^\dagger - b_q b_{-q}]\right). \tag{B5}$$

Here $\tanh(2\varphi) = (1-g^2)/(1+g^2)$. The energy ϵ_p in Eqs. (B3) and (B4) is now the energy of plasmons $\epsilon_p = s|p|$ in a LL.

APPENDIX C

In this appendix we list the analytical expressions for the integrals of the Γ functions appearing in the evaluation of the LL thermopower:

$$\int_{-\infty}^{\infty} dx |\Gamma(\alpha+ix)|^2 |\Gamma(\beta+ix)|^2 = 2\pi \frac{\Gamma^2(\alpha+\beta)\Gamma(2\alpha)\Gamma(2\beta)}{\Gamma(2(\alpha+\beta))}, \tag{C1}$$

$$\int_{-\infty}^{\infty} dx \left\{ x \left| \Gamma\left(\alpha + \frac{i}{2}(x+z) \right) \right|^2 \left| \Gamma\left(\beta + \frac{i}{2}(x-z) \right) \right|^2 \right\} = \frac{2\pi z}{(\alpha+\beta)} \frac{\Gamma^2(\alpha+\beta+iz)\Gamma(2\alpha)\Gamma(2\beta)}{\Gamma(2(\alpha+\beta))}, \tag{C2}$$

$$\int_{-\infty}^{\infty} dx \{ x^2 |\Gamma(\alpha+ix)|^2 |\Gamma(\beta+ix)|^2 \} = \frac{2\pi\alpha\beta}{(2\alpha+2\beta+1)} \frac{\Gamma^2(\alpha+\beta)\Gamma(2\alpha)\Gamma(2\beta)}{\Gamma(2(\alpha+\beta))}. \tag{C3}$$

The first integral can be found in the tables of integrals (see, e.g., Ref. 30). The two other integrals are readily derived from Eq. (C1).

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