

Aharonov-Bohm and Aharonov-Casher tunneling effects and edge states in double-barrier structures

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The simultaneous occurrence of Aharonov-Bohm (AB) and Aharonov-Casher (AC) effects due to edge states in double-barrier two-dimensional wires formed by an electrostatic confinement potential, in the quantum Hall effect regime, is discussed. The AC effect is manifested via a shift of the AB conductance oscillations, and a method for measurement of the effect is proposed.

Quantum coherent effects have been intensely studied for a long time. The most well known of them is the Aharonov-Bohm (AB) effect due to sensitivity of the phase of the electron wave function to the vector potential field, which creates magnetic flux.¹ The additional shift of the phase of the wave function (the AB phase) when an electron moves along a closed trajectory [Fig. 1(a)] is given by the expression

$$\Phi_{AB} = \frac{e}{\hbar c} \oint \mathbf{A} \cdot d\mathbf{l} \equiv 2\pi \frac{\phi}{\phi_0}, \tag{1}$$

where ϕ is the magnetic flux enclosed by the trajectory and $\phi_0 = hc/e$ is the flux quantum.

Another example of a quantum coherence effect is the Aharonov-Casher (AC) effect.² The latter, being dual to the AB effect, is due to interaction between the particle's magnetic moment μ and an electric field \mathbf{E} . The phase shift in this case (the AC phase) is given by

$$\Phi_{AC} = \frac{1}{\hbar c} \oint (\mathbf{E} \times \boldsymbol{\mu}) \cdot d\mathbf{l}. \tag{2}$$

In the case of a Coulomb electric field, which may be created, for example, by a charge line with charge density τ [Fig. 1(a)], $\mathbf{E} = 2\tau\mathbf{r}/r^2$, we have for the AC phase the following expression:

$$\Phi_{AC} = 2\pi \frac{F}{F_0}, \tag{3}$$

where $F = \int E_n d\mathbf{l} = 4\pi\tau$ is the electric flux through the loop trajectory and $F_0 = hc/\mu$ is the "electric flux quantum." Note that in this case the AC phase is a topological one as in the AB effects, when the magnetic flux ϕ is created by a solenoid. Both phases depend on the magnitudes of the fluxes (for the AC phase on charge density) and do not depend on the shape and size of the trajectory. An important difference between the two phenomena should be emphasized. The AB effect originates from charge current-vector potential interaction and has a force-free nature. Thus, it may take place even in a magnetic-field-free region. The AC effect results from spin-orbit interaction, so it is necessary to create a real electric field for its existence.³ Both the AB effect and the AC one were first observed for electron⁴ and neutron⁵ beams, in vacuum.

There are various solid-state realizations of the AB and AC effects. The AB effect is manifested in solids in oscil-

latory dependence of thermodynamic and kinetic characteristics on magnetic flux, with a flux quantum period. This phenomenon occurs both in double-connected systems (pure⁶ and dirty^{7,8} cylinders and rings, see Refs. 9–12) and in simply connected ones.¹³ In the latter case the AB oscillations are due to electron edge states localized near the surface and forming effectively a double-connected (ring) geometry. The concept of edge states (whispering-gallery states) was used for study of the AB effect in solid cylinders (microwires) placed in a weak longitudinal magnetic field.¹³ This geometry (Bi microcylinders) allowed the experimental observation of the AB effect.¹⁴ Later the AB effect due to similar edge states was studied in quantum dots,¹⁵ point contact,^{16,17} and a single-barrier structure¹⁸ in a strong magnetic field (see Ref. 19).

The AC effect reveals itself in solids also, through a specific dependence of thermodynamic and transport properties on electric flux (the AC phase). So far, different theoretical aspects of this effect have been dis-

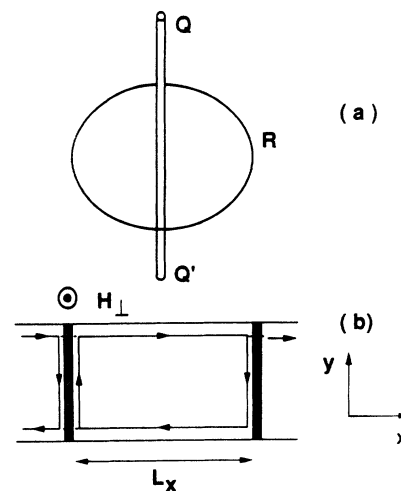


FIG. 1. (a) Schematic of the Aharonov-Bohm (AB) and Aharonov-Casher (AC) effects. R is an electron trajectory and QQ' is a source of flux created by a solenoid (AB effect) or by a charged fiber (the AC effect). (b) Schematic of a two-dimensional quantum wire with two barriers (hatched) in the geometry of the quantum Hall effect (H_{\perp} is in the direction into the page). Arrows denote the directions of motion of the electrons via edge states.

cussed only in the context of double-connected metallic,²⁰ magnetic,²¹ semiconducting,²² and ³He-*A* (Ref. 23) systems (see also Ref. 24 about spin-orbit interaction in mesoscopic systems). The purpose of the present paper is to suggest measurement of the AC effect in simply connected systems in the quantum Hall effect regime.

The geometry of the transport problem considered here is shown in Fig. 1(b). A two-dimensional wire (long in the *x* direction and finite in the *y* direction) with two tunneling barriers (along the *x* direction) is placed in a strong magnetic field *H* normally oriented to the surface (i.e., geometry of the quantum Hall effect). In this case the magnetic moments of all the electrons are oriented along the magnetic field. The distance between the barriers along the *x* direction, *L_x*, is supposed to be small compared to the phase-breaking length $l_\phi = \sqrt{D\tau_\phi}$ and the normal metal coherence length

$$l_T = \min \left\{ \frac{\hbar v_F}{\pi T}, \left(\frac{\hbar D}{T} \right)^{1/2} \right\},$$

where $D \sim v_F l$ is the electron diffusion coefficient, v_F is the Fermi velocity, l the elastic mean free path of the electrons, τ_ϕ is the phase-breaking time, and T is the temperature.

Restriction of the dimension of the wire in the *y* direction may be achieved via a confinement potential *V*, usually of electrostatic nature. In this case an electric field is applied in the plane of the wire. Electric and magnetic fields cause motion of electrons along the boundaries via edge states.¹⁹ Because electrons in edge states at opposite sides of the wire move in opposite directions, backscattering (reflection) from the barriers requires tunneling jumps from one side of the wire to another. This leads to formation of a closed path of the electrons between the barriers [see Fig. 1(b)] that results in the simultaneous appearance of the AB and AC phases.

We will describe the electric transport in the wire in terms of the transmission $t_{1(2)}$ and the reflection $r_{1(2)}$ amplitudes of barriers 1 and 2. The total transmission probability (\mathcal{T}) through the double-barrier structure for a single edge channel, including contributions from all closed electron trajectories between the barriers calculated to all orders of reflections from the barriers, has the form (com-

pare with Ref. 19)

$$\mathcal{T} = \left| \frac{t_1 t_2}{1 - r_1 r_2 \exp(i\phi_{AB} + i\phi_{AC})} \right|^2. \quad (4)$$

The AB and AC phases may be expressed in terms of Eqs. (1) and (3) with fluxes depending on sample geometry, which determines the electron trajectory; $\phi = H_1 S$ (S is the area enclosed by the electron trajectory) and $F = 2EL_x$.

In order to estimate the AC phase shift in Eq. (4) let us consider the electric field associated with confinement of the electrons in the *y* direction

$$E = -\frac{1}{e} \frac{\partial V}{\partial y} \sim -\frac{1}{e} \frac{\epsilon_F}{\Delta y}$$

and the magnetic moment $\mu = g\mu_B$, where ϵ_F is the Fermi energy, Δy is the length scale of substantial variation of the confining potential, g is the g factor, and μ_B is the Bohr magneton. This yields

$$\phi_{AC} \sim g \frac{\epsilon_F}{mc^2} \frac{L_x}{\Delta y} \sim g \left(\frac{v_F}{c} \right)^2 \frac{L_x}{\Delta y}. \quad (5)$$

For semiconducting wires with large values of the g factor (~ 100 , see Refs. 22 and 23) and for typical values of the ratio $v_F/c \sim 10^{-2}$, the system characteristic length ratio $L_x/\Delta y$ may be chosen to be of the order 10^2 , yielding an AC phase of the order of unity. To observe the AC phase it may be more convenient to place an additional magnetic field H_{\parallel} in the plane of the wire in the *y* direction. If this field is much stronger than the Hall magnetic field (H_1), the magnetic moments of all the electrons will be oriented toward the plane of the wire. This leads to effective cancellation of the AC phase [see Eq. (2)] and, consequently, to a shift of the AB oscillations [that is, dependence of \mathcal{T} on the AB phase, see Eq. (4)]. Thus, measurements with and without the magnetic field H_{\parallel} would allow determination of ϕ_{AC} . In this context we note that, using this method of measurement, experimental conditions for which $\phi_{AC} \sim 1$ are not essential.

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